

Pion and Quark Annihilation Mechanisms of Dilepton Production in Relativistic Heavy Ion Collisions

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Abstract

We revise the $\pi^+\pi^-$ and $q\bar{q}$ annihilation mechanisms of dilepton production during relativistic heavy-ion collisions. We focus on the modifications caused by the specific features of intramedium pion states rather than by medium modification of the ρ -meson spectral density. The main ingredient emerging in our approach is a form-factor of the multi-pion (multi-quark) system. Replacing the usual delta-function the form-factor plays the role of distribution which, in some sense, "connects" the 4-momenta of the annihilating and outgoing particles. The difference between the c.m.s. velocities attributed to annihilating and outgoing particles is a particular consequence of this replacement and results in the appearance of a new factor in the formula for the dilepton production rate. We obtained that the form-factor of the multi-pion (multi-quark) system causes broadening of the rate which is most pronounced for small invariant masses, in particular, we obtain a growth of the rate for the invariant mass below two masses of the annihilating particles.

Since leptons do not interact practically with the highly excited nuclear matter, they leave the reaction zone after their creation in a relativistic nucleus-nucleus collision without further rescattering. That is why, the dileptons (e^+e^- and $\mu^+\mu^-$ pairs) observed in high-energy heavy ion collisions secure an excellent opportunity for obtaining information on the initial state and the evolution of the system created in a collision. The enhancement with an invariant mass of $200 \div 800$ MeV observed by the CERES collaboration [1, 2] in the production of dileptons has received recently a considerable attention and has been studied in the framework of various theoretical models (for the review, see Ref. [3]). It was found that a large part of the observed enhancement is due to the medium effects (see Refs. [4, 5] and references therein). Meanwhile, pion annihilation is the main source of dileptons which come from the hadron matter [6, 7]. That is why, the proper analysis of the dilepton spectra obtained experimentally gives important observables which probe the pion dynamics in the dense nuclear matter that exists at the early stage of the collision. The purpose of the present letter is to look once more on the $\pi^+\pi^-$ annihilation mechanism of dilepton production from the hadron plasma by accounting the medium-induced modifications of the dilepton spectrum. In order to do this, we concentrate on the modifications which are due rather to intramedium pion states, than on the discussion of a modification of the ρ -meson spectral density. In accordance with our suggestions, the main features of a pion wave function follow from the fact that pions live a finite time in the

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system where they can take part in the annihilation reaction. As a consequence, the off-shell effects give an appreciable contribution to the features of the annihilation process specifically in the region of low invariant masses. Moreover, if pions are the entities of a local subsystem, then the spatial structure of the pion states is far from a plane-wave one and this also gives the essential contribution to the features of the dilepton spectrum.

To carry out the outlined program, we assume that the pion liquid formed after the equilibration exists in a finite volume, and the confinement of pions to this volume is a direct consequence of the presence of the dense hadron environment which prevents the escape of pions during some mean lifetime τ . The same can be assumed concerning a hot system of quarks which are confined to a quark-gluon droplet. We sketch the proper geometry in Fig. 1. The small circle of radius R represents a subsystem of pions which is in a local thermal equilibrium and moves with the collective velocity \mathbf{v} . If the fireball size is of order of $R_0 \propto 6 \div 10$ fm, then the mean size of the small subsystem is of order of $R \propto 1 \div 5$ fm. So, we assume the system of pions (quarks) produced in high-energy heavy-ion collisions is effectively bounded in a finite volume. The pion (quark) wave functions $\varphi_\lambda(x)$, where λ is a quantum number, satisfy the proper boundary conditions and belong to the complete set of functions. For instance, the stationary wave functions may be taken as the solutions of the Klein-Gordon equation $(\nabla^2 + k^2) \varphi_\lambda(\mathbf{x}) = 0$, where $k^2 = E^2 - m^2$, which satisfy the Dirichlet boundary condition on the surface S : $\varphi_\lambda(\mathbf{x})|_S = 0$. For the box boundary, we get $\varphi_{\mathbf{k}}(\mathbf{x}) = \sqrt{8/V} \prod_{i=1}^3 \theta(L_i - x_i) \theta(x_i) \sin(k_i x_i)$, where $V = L_1 L_2 L_3$ is the box volume, $\lambda \equiv \mathbf{k} = (k_1, k_2, k_3)$, and components of the quasi-momentum run through the discrete set $k_i = \pi n_i / L_i$ with $n_i = 1, 2, 3, \dots$. For the spherical geometry, the normalized solutions are written as $\varphi_{klm}(\mathbf{r}) = \theta(R - r) (2/r)^{1/2} J_{l+1/2}(kr) Y_{lm}(\vartheta, \phi) / R J_{l+3/2}(kR)$, where $\lambda = (k, l, m)$. Next, the field operators $\hat{\varphi}(x)$ corresponding to the pion field should be expanded in terms of these eigenfunctions, i.e.

$$\hat{\varphi}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_{\mathbf{k}}} [a(\mathbf{k})\varphi_{\mathbf{k}}(x) + b^+(\mathbf{k})\varphi_{\mathbf{k}}^*(x)] , \quad (1)$$

where $a(\mathbf{k})$ and $b(\mathbf{k})$ are the annihilation operators of positive and negative pions, respectively.

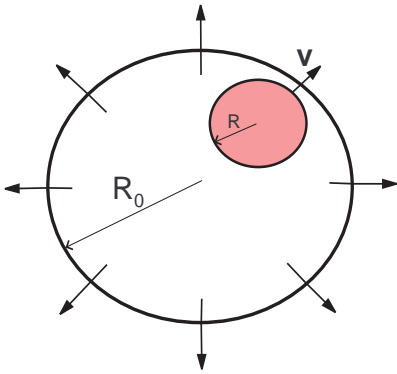


Fig. 1. Sketch of an expanding fireball. The small circle of radius R represents the subsystem of pions which is in a local thermodynamic equilibrium and moves with the collective velocity \mathbf{v} .

On the other hand, the states corresponding to confined particles can be written in a common way as $\varphi_{\mathbf{k}}(\mathbf{x}) = \sqrt{\rho(\mathbf{x})/V} \Phi_{\mathbf{k}}(\mathbf{x})$, where $\rho^{1/2}(\mathbf{x}) = \prod_{i=1}^3 [\theta(L_i - x_i) \theta(x_i)]$ for a box and $\rho^{1/2}(\mathbf{x}) = \theta(R - r)$ for a sphere, respectively. The function $\rho(\mathbf{x})$ presents the information about the geometry of a reaction region or cuts out the volume where the pions (quarks) can annihilate. Hence, for the evaluation of S -matrix elements wave functions $\varphi_{\mathbf{k}}(x)$ should be taken as the pion *in*-states once annihilating pions belong to finite system. The amplitude of the pion-pion annihilation to a lepton pair in the first non-vanishing approximation is calculated via the chain $\pi^+ \pi^- \rightarrow \rho \rightarrow \gamma^* \rightarrow \bar{l} l$, where the ρ -meson appears as an intermediate state in accordance with the vector meson dominance. The matrix element of the reaction is $\langle \text{out} | S | \text{in} \rangle = - \int d^4x_1 d^4x_2 \langle \mathbf{p}_+, \mathbf{p}_- | T [H_I^\pi(x_1) H_I^l(x_2)] | \mathbf{k}_1, \mathbf{k}_2 \rangle$, where $H_I^\pi(x) = -e j_\mu^\pi(x) A^\mu(x)$ and $H_I^l(x) = -e j_\mu^l(x) A^\mu(x)$. It is remarkable that the pion density $\rho(\mathbf{x})$ appears as a factor of the pion current. Indeed,

$$j_\mu^\pi(x) = -i \hat{\varphi}(x) \overleftrightarrow{\partial}_\mu \hat{\varphi}^+(x) = \frac{\rho(\mathbf{x})}{V} \left[-i \hat{\Phi}(x) \overleftrightarrow{\partial}_\mu \hat{\Phi}^+(x) \right] , \quad (2)$$

where the field operator $\hat{\Phi}(x)$ is defined in the same way as that in (1) with the functions $\varphi_{\mathbf{k}}(\mathbf{x})$

replaced by $\Phi_{\mathbf{k}}(\mathbf{x})$. Because of this factorization, after the integration over the vertex x the density $\rho(\mathbf{x})$ automatically cuts out the volume, where the $\pi^+\pi^-$ annihilation reaction is running. At the same time, this means that the density $\rho(\mathbf{x})$ determines the volume of quantum coherence, i.e. just the particles from this spatial domain are capable to annihilate one with another and make contribution to the amplitude of the reaction. To obtain the overall rate, it is necessary then to sum up the rates from every coherent domain of the fireball.

For the sake of simplicity, we assume that the pion states can be approximately represented as $\varphi_{\mathbf{k}}(x) = \sqrt{\rho(x)/V} e^{-ik \cdot x}$. Here, $\rho(x)$ is the 4-density of pions in the volume V (marked by radius R in Fig. 1), where pions are in a local thermodynamic equilibrium. In essence, this approximation considers just one mode of the wave function $\Phi_{\mathbf{k}}(\mathbf{x})$. We expect that this ansatz reflects the qualitative features of the pion states in a real hadron plasma, and, below, the main consequences of the contraction of particle states in the dense environment are considered.

A simple calculation immediately shows that the S -matrix element is proportional to the Fourier-transformed pion density $\rho(x)$, i.e. $\langle \text{out} | S | \text{in} \rangle \propto \rho(k_1 + k_2 - p_+ - p_-)$, where k_1 and k_2 are the 4-quasi-momenta of the initial pion states and p_+ and p_- are the 4-momenta of the outgoing leptons. This means that the form-factor of the pion source $\rho(k)$ stands here in place of the delta function which appears in the standard calculations, i.e. $(2\pi)^4 \delta^4(K - P) \rightarrow \rho(K - P)$, where $K = k_1 + k_2$ and $P = p_+ + p_-$ are the total (quasi-) momenta of pion and lepton pairs, respectively. An immediate consequence of this is a breaking down of the energy-momentum conservation in the s -channel of the reaction, which means that the total momentum K of the pion pair is no longer equal exactly to the total momentum P of the lepton pair. The physical interpretation of this fact is rather obvious: the effect of the hadron environment on the pion subsystem which prevents the escape of pions from the fireball can be regarded during the time span τ as the influence of an external nonstationary field. The latter, as known, breaks down the energy-momentum conservation. From now, the squared form-factor $|\rho(K - P)|^2$ of the pion system plays the role of a distribution which in some sense "connects" in s -channel the annihilating and outgoing particles instead of δ -function. Indeed, the number $N^{(\rho)}$ of produced lepton pairs from a finite pion system related to an element of the dilepton momentum space, reads

$$\left\langle \frac{dN^{(\rho)}}{d^4P} \right\rangle = \int d^4K |\rho(K - P)|^2 \left\langle \frac{dN}{d^4K d^4P} \right\rangle, \quad (3)$$

where

$$\begin{aligned} \left\langle \frac{dN}{d^4K d^4P} \right\rangle &= \int \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2} \delta^4(k_1 + k_2 - K) f_{\text{th}}(E_1) f_{\text{th}}(E_2) \\ &\times \int \frac{d^3p_+}{(2\pi)^3 2E_+} \frac{d^3p_-}{(2\pi)^3 2E_-} \delta^4(p_+ + p_- - P) |A_0(k_1, k_2; p_+, p_-)|^2. \end{aligned} \quad (4)$$

Here, $E_i = \sqrt{m^2 + \mathbf{k}_i^2}$, $i = 1, 2$ for pions and $i = +, -$ for leptons, respectively. By the broken brackets, we denote the thermal averaging over the pion quasi-momentum space with the help of the thermal distribution function $f_{\text{th}}(E)$. To obtain eq.(3), we represent the amplitude $A_\rho = \langle \text{out} | S^{(2)} | \text{in} \rangle$ of the reaction under consideration as

$$A_\rho(k_1, k_2; p_+, p_-) = \rho(k_1 + k_2 - p_+ - p_-) A_0(k_1, k_2; p_+, p_-). \quad (5)$$

We note that not only the form-factor $\rho(K - P)$ contains information about the pion system. The amplitude A_0 carries new important features as well, which are related to the violation of the energy-momentum conservation in the s -channel. Indeed, the pion-pion c.m.s. moves with the velocity

$\mathbf{v}_K = \mathbf{K}/K_0$, whereas the lepton-pair c.m.s. moves with the velocity $\mathbf{v}_P = \mathbf{P}/P_0$. Hence, these two center-of-mass systems are "disconnected" now, that is why any quantity should be Lorentz-transformed when transferred from one c.m.s. to another. A particular consequence of this is the appearance of the correction factor $\left[1 + \frac{1}{3} \left(\frac{(P \cdot K)^2}{P^2 K^2} - 1 \right)\right]$ in the formula for the dilepton production rate (for details see [8]).

Concerning the physical meaning of eq. (3), we note that one can regard it as the averaging of the random quantity $\left\langle \frac{dN}{d^4 K d^4 P} \right\rangle$ with the help of the distribution function $|\rho(K - P)|^2$ centered around the mean value P . In this sense, the hadron medium holding pions in a local spatial region for some time, which is expressed as the local pion distribution $\rho(x)$, plays the role of an environment randomizing the pion source. This randomization is a purely quantum one in contrast to the thermal randomization of the multi-pion system which is already included to the quantity $\left\langle \frac{dN}{d^4 K d^4 P} \right\rangle$.

In order to transform the distribution of the number of created lepton pairs over the dilepton momentum space to the distribution over invariant masses, one has to perform additional integration using $\langle dN^{(\rho)}/d^4 P \rangle$ from (3), i.e. $\left\langle \frac{dN^{(\rho)}}{dM^2} \right\rangle = \int \frac{d^3 P}{2P_0} \left\langle \frac{dN^{(\rho)}}{d^4 P} \right\rangle$, where $P_0 = \sqrt{M^2 + \mathbf{P}^2}$. Taking this together we obtain

$$\begin{aligned} \left\langle \frac{dN^{(\rho)}}{dM^2} \right\rangle = & \frac{\alpha^2}{3(2\pi)^8} \left(1 - \frac{4m_e^2}{M^2}\right)^{1/2} \left(1 + \frac{2m_e^2}{M^2}\right) |F_\pi(M^2)|^2 \int \frac{d^3 P}{2P_0} \int d^4 K \frac{K^2}{M^2} |\rho(K - P)|^2 e^{-\beta K_0} \\ & \times \left(1 - \frac{4m_\pi^2}{K^2}\right)^{3/2} \left[1 + \frac{1}{3} \left(\frac{(P \cdot K)^2}{M^2 K^2} - 1 \right)\right] \theta(K_0) \theta(K^2 - 4m_\pi^2), \quad (6) \end{aligned}$$

where $F_\pi(M^2)$ is the ρ -meson form-factor, and we take the Boltzmann distribution $f_{\text{th}}(E) = \exp(-\beta E)$ with $\beta = 1/T$ as inverse temperature. By the presence of the θ -functions (last two factors on the r.h.s. of (6)), we would like to stress that the invariant mass of a pion pair $M_\pi = \sqrt{K^2}$ is not less than two pion masses. On the other hand, possible finite values of the distribution $\left\langle \frac{dN^{(\rho)}}{dM^2} \right\rangle$ below the two-pion mass threshold can occur just due to the presence of the pion system form-factor $\rho(K - P)$.

The factor in the square brackets on the r.h.s. of (6) is a correction which is due to the Lorentz transformation of the quantity $(\mathbf{k}_1 - \mathbf{k}_2)^2$ from the dilepton c.m.s. to the pion-pion c.m.s. This factor gives a remarkable contribution to the dilepton spectrum for invariant masses below the two-pion mass value. Its influence is especially pronounced for e^+e^- production (see [8]).

To clarify the effects under investigation as much as possible, for particular evaluations we take as a model of pion system the Gaussian distribution of the particles in space and the Gaussian decay of the system of pions:

$$\rho(x) = \exp\left(-\frac{t^2}{2\tau^2} - \frac{\mathbf{r}^2}{2R^2}\right). \quad (7)$$

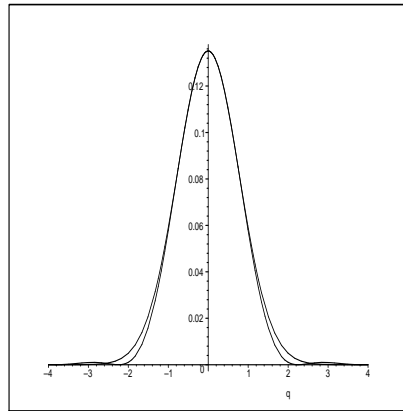


Fig. 2. Comparison of the Gaussian and θ -function form-factors $|\rho(\mathbf{q})|^2/V$.

Meanwhile, it can be another choice of the model function for the pion source. Indeed, one can choose for instance a geometry with sharp boundaries which are determined by the θ -functions. To elucidate this point we compare two formfactors (normalized to the unit volume) which correspond to the Gaussian distribution

$\rho_g(\mathbf{r}) = e^{-\frac{\mathbf{r}^2}{2R^2}}$ and to the θ -function distribution $\rho_\theta(\mathbf{r}) = \theta(R - |\mathbf{r}|)$ (see Fig. 2). Only a slight difference

between these formfactors is seen and, therefore, the choice of pion source distribution doesn't affect much the dilepton production rate. In what follows, we dwell on the Gaussian distribution.

To be closer to the quantities measured in experiment, we evaluate the rate related to the proper rapidity window

$$\frac{dR}{dM dy} = 2\pi M \frac{1}{\Delta y} \int_{y_{\min}}^{y_{\max}} dy \int_{P_{\perp \min}}^{\infty} dP_{\perp} P_{\perp} \frac{dN}{d^4x d^4P},$$

where $\tanh y = P^3/P^0$, $P_{\perp}^2 = (P^1)^2 + (P^2)^2$. The results of evaluation of the production rates $dR_{e^+e^-}^{(\rho)}/dM dy$ and $dR_{\mu^+\mu^-}^{(\rho)}/dM dy$ for electron-positron and muon-muon pairs, respectively, in pion-pion annihilation are depicted in Fig. 3. Different curves correspond to the different "spatial sizes" R and different "lifetimes" τ of a hot pion system at the temperature $T = 180$ MeV. Notice that the production rate in a finite small pion system differs from the rate in a infinite pion gas (solid curve) where pion *in*-states can be taken as plane waves. The deviation bigger when the parameters R and τ are smaller. Of course, this is a reflection of the uncertainty principle which is realized by the presence of the distribution $|\rho(K - P)|^2$ as the integrand factor in (3). Basically, the presence of the form-factor of the multi-pion system will result in a broadening of the rate for small invariant masses $M \leq 800$ MeV/c² which is wider at the smaller parameters R and τ . This seems natural because the quantum fluctuations of the momentum are more pronounced in smaller systems. We emphasize as well that the behavior of the curves in Fig. 3, which correspond to a finite system, has a similar tendency to the CERES data [1, 2].

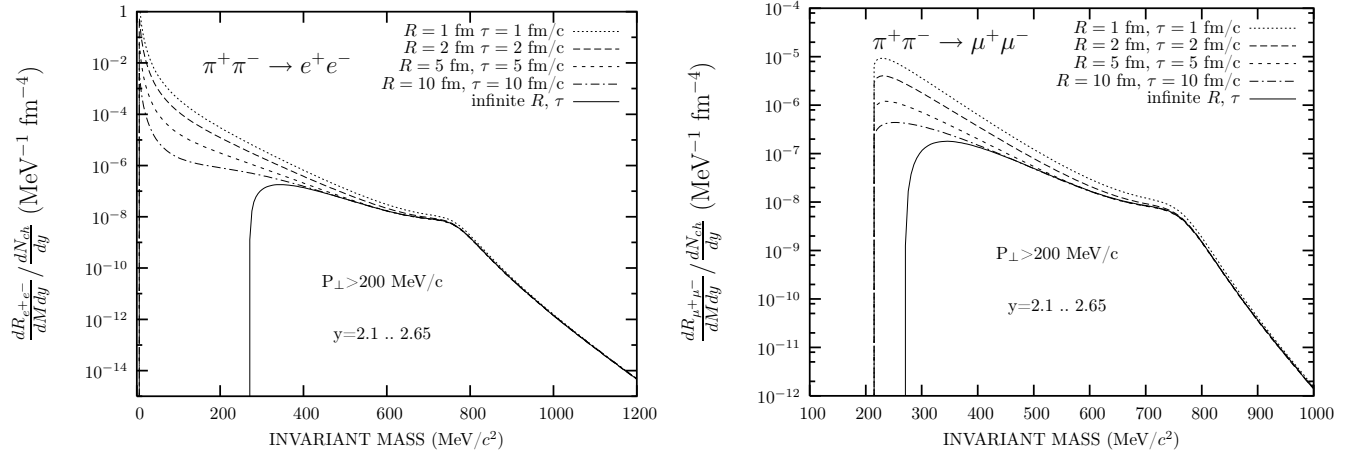


Fig. 3. Rates of electron-positron (left panel) and muon-muon (right panel) productions in pion-pion annihilation in a small finite system, $T = 180$ MeV.

For comparison, we present in Fig. 4 the results of evaluation of the rate $dR_{e^+e^-}^{(\rho)}/dM dy$ of electron-positron pair production in quark-antiquark annihilation in a QGP drop. The evaluations was made under the same assumptions as for pion-pion annihilation. As in the previous case, one can see an increase in the rate with a decrease in the invariant mass up to two electron masses. This real threshold is close to the total mass of annihilating quarks $M = 2m_q \approx 10$ MeV/c². The different behavior of the rate for small parameters R and τ in the region of small invariant masses $M \leq 500$ MeV/c², as compared to the rate for infinite parameters $R = \infty$, $\tau = \infty$, is due to an increase of quantum fluctuations which are evidently bigger for a smaller size of the QGP drop.

One more important result is worth to note: the enhancement of the dilepton production rate for the low invariant mass region is much more sensitive to the variation in the spatial size of a many-particle (pion, quark) system than to that in the system lifetime (see Fig. 4, right panel). This

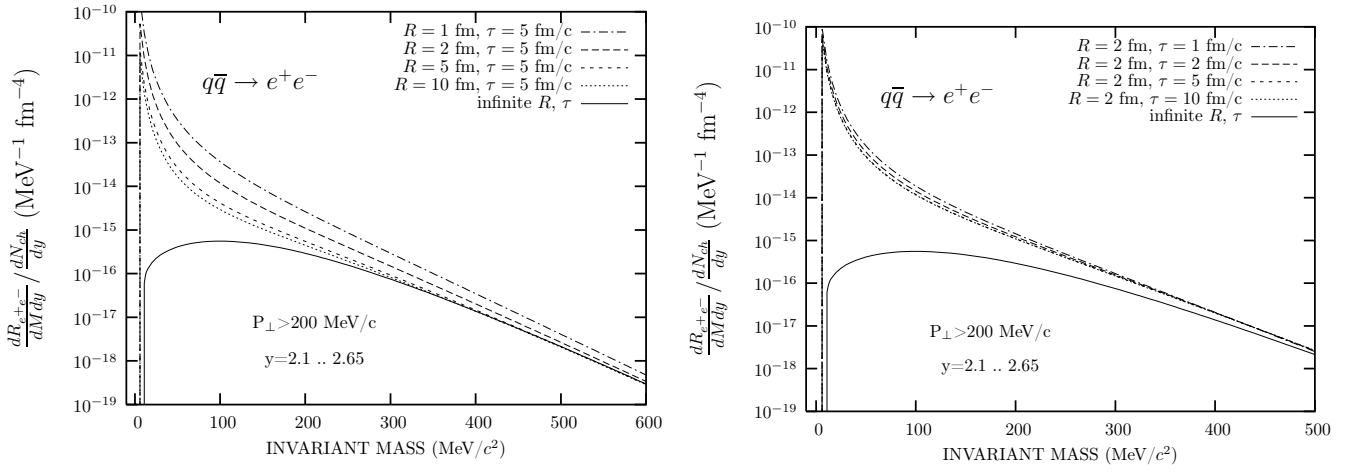


Fig. 4. Electron-positron production rates in quark-antiquark annihilation in a hot QGP drop, $T = 180$ MeV.

fact can be explained with the use of the particle density taken in a particular form (7). Indeed, for the system which is confined in three spatial dimensions, each of the three spatial densities in (7), for instance $\exp(-x_i^2/R^2)$ with $i = 1, 2, 3$, corresponds to one integration in (1) over a component of the pion-pair total momentum K_i . Hence, each of the three integrations on the r.h.s. of (3) involves a factor which is responsible for quantum fluctuations in its own spatial dimension; for instance, one of the three factors is $(2\pi)^{1/2}R \cdot \exp(-R^2K_i^2)$. Meanwhile, the spectral function which is responsible for the system lifetime finiteness, for instance $(2\pi)^{1/2}\tau \cdot \exp(-\tau^2K_0^2)$ as in our approach, emerges as an integrand factor just once during the integration over K_0 . That is why the 3-dimensional variations of the spatial size of a many-particle system, as seen in Fig. 4, influence the shape of the dilepton production rate much stronger than the variations of the lifetime of a small system.

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